Microwave Scattering in the Subohmic Spin-Boson Systems of Superconducting Circuits

TSUYOSHI YAMAMOTO AND TAKEO KATO
INSTITUTE FOR SOLID STATE PHYSICS, THE UNIVERSITY OF TOKYO

ABSTRACT

We proposed an experimental setup for the realization of quantum phase transitions in superconducting circuits composed of a qubit and transmission lines. The microwave reflection at the qubit shows distinctive frequency behavior that originates from the nature of quantum critical phenomena. The present study provides a useful experimental platform for the observation of quantum critical phenomena in qubit-photon hybrid systems.

INTRODUCTION

In classical systems, phase transitions are induced by thermal fluctuation as the ice melts into the water at 0˚C. In contrast, quantum systems may possess zero-temperature phase transitions induced by quantum fluctuation, which is called a quantum phase transition (QPT). Near QPT, one may observe various quantum critical phenomena (QCP) via temperature- or frequency-dependence of physical quantities. QCP have been a hot topic in condensed matter physics and has been studied mainly in the magnetic ordering of solids for a long time [1]. To realize QCP in solids experimentally, one needs to tune experimental parameters such as pressure. Tuning these parameters usually makes the detailed analysis of QCP difficult. Recent rapid development in the fabrication of superconducting qubits has enabled us to prepare well-controlled qubit-photon hybrid systems composed of qubits and microwave cavities [2]. In our recent paper [3], we have pursued the possibility of observation of QCP in this ideal physical system.

Subohmic Spin-Boson Model

A system composed of a superconducting qubit and superconducting circuits is described by the spin-boson model, which is a model of a two-state system coupled to reservoirs described by harmonic oscillators. The qubit-reservoir coupling is characterized by the spectral density function $I(\omega)$, which is a function of the frequency of harmonic oscillators. The low-frequency part of the spectral density is assumed to have power-law behavior $I(\omega) = A\omega^s$, where the exponent s characterizes the properties of the spin-boson model. For the subohmic case ($0 < s < 1$), the spin-boson model is known to display a QPT [4] because the qubit-reservoir coupling at low frequencies is strong. When the prefactor A exceeds a critical value, coherent superposition of a qubit is completely broken, leading to a QPT from a singlet ground state to a doublet one. However, so far, the experimental realization of the subohmic spin-boson model has not been considered except for the special case of $s = 1/2$ [5].

Quantum Critical Phenomena in the Superconducting Circuit

There are several types of superconducting qubits such as charge qubits, flux qubits, phase qubits, and transmons. We focus on a charge qubit, in which two low-energy charge states in a small superconductor (a Cooper-pair box) are assigned to a bit. In order to observe QPT, it is necessary to couple a charge qubit to a superconducting circuit with subohmic spectral density. In this study [3], we propose an RLC array circuit composed of resistances, inductances, and capacitances as shown in Fig. 1.

In contrast to the previous proposal [5], we assume that the circuit elements are not constant but spatial-dependent. One can show that this superconducting

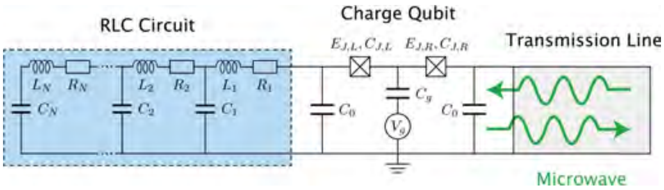

Fig. 1: Superconducting circuit described by the subohmic spin-boson model.

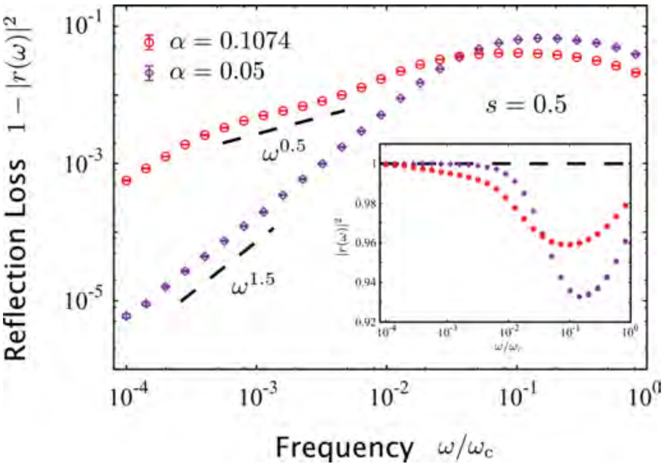

Fig. 2: Numerical results for the microwave reflection loss $1-|r(\omega)|^2$ as a function of the frequency for $s = 0.5$. α is the dimensionless qubit-reservoir coupling. Red and purple plots represent the critical point ($\alpha = 0.1074$) and below it ($\alpha = 0.05$), respectively.

circuit plays a role in the subohmic environment with an arbitrary exponent of the spectral density s. In order to probe QCP, we also consider an additional transmission line weakly coupled to the target hybrid system composed of a charge qubit and an RLC array circuit (see Fig. 1) and consider microwave reflection loss at the target. We theoretically calculate the reflection loss as a function of the frequency (see Fig. 2) using the quantum Monte Carlo method. As seen in Fig. 2, we observe the distinctive power-law frequency dependence, ω^{1-s} for $s = 0.5$, when the dimensionless qubit-reservoir coupling α ($\propto A$) is set to a critical value, 0.1074. This behavior, characteristic of QCP, is consistent with the renormalization group analysis. We note that if the qubit-reservoir coupling is taken away from the critical point ($\alpha = 0.05$), the frequency dependence of the loss becomes ω^{1+s}, which is characteristic of the singlet ground state. In the present proposal for observing QCP in superconducting circuits, the critical exponent is controllable because the exponent of a subohmic spectral density can be changed. So far, the subohmic spin-boson model has been studied theoretically as a toy model of QCP and has not considered to be realistic for experiments. The present study will, therefore, encourage to observe QCP using superconducting circuits experimentally in the near future.

Acknowledgements: The authors thank the Supercomputer Center, the Institute for Solid State Physics, the University of Tokyo for the use of the facilities. T.K. was supported by JSPS Grants-in-Aid for Scientific Research (Nos. JP24540316 and JP26220711).

References

[1] S. Sachdev, Quantum Phase Transition (Cambridge University Press, Cambridge, England, 1999).
[2] P. Forn-Díaz et al., Nat. Phys. 13, 39 (2017); L. Magazzù et al., Nat. Commun. 9, 1403 (2018).
[3] T. Yamamoto and T. Kato, J. Phys. Soc. Jpn. 88, 094601 (2019).
[4] A. J. Leggett et al., Rev. Mod. Phys. 59, 1 (1987); U. Weiss, Quantum Dissipative Systems (World Scientific, Singapore, 1999) 4th ed.
[5] N. H. Tong and M. Vojta, Phys. Rev. Lett. 97, 016802 (2006).

Reference 3 was published in the August 2019 issue of the *Journal of the Physical Society of Japan* as an Editors' Choice article.

Tsuyoshi Yamamoto is a PhD student at the Department of Physics, Graduate School of Science, The University of Tokyo.

Takeo Kato is an associate professor at the Institute for Solid State Physics, The University of Tokyo. He earned his DSci from the University of Tokyo in 1999. After working as a JSPS (Japan Society for the Promotion of Science) Research Fellow and as a lecturer at Osaka City University, he became an associate professor at The University of Tokyo in 2004.