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Supergravity

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ABSTRACT

Supergravity has been at the heart of many developments in theoretical high-energy physics in the last 40 years. It is interconnected with String Theory, with contemporary mathematical developments and is at the base of models as well in Particle Physics as in Cosmology. Despite its 40 years, new insights in the structure of Supergravity are still obtained. This includes the building of new models and the discovery of new solutions of its dynamical equations, used for research in black holes, dualities with strings and branes and holography. The inventors of Supergravity recently obtained the breakthrough prize in fundamental physics for the influence that they have had on theoretical physics.

1 INTRODUCTION

Since 100 years we know that gravity is best described by general relativity, where gravity is the effect of the warping of spacetime by massive objects. Particle physics, on the other hand, considers the forces as mediated by force-carrying particles and ignores gravity. The central challenge of theoretical physics is the question how to join the physics of the very small with the physics of the very big, where mainly effects of gravity are visible.

All known particles are described within the framework known as the Standard Model of Particle Physics. The Standard Model has been incredibly successful, describing very accurately the experimental results of the Large Hadron Collider (LHC) at CERN, Geneva. It predicted the existence of the Higgs boson, which was discovered in 2012. It describes three of the four fundamental forces: electromagnetism and the so-called strong and weak forces acting inside atomic nuclei. However, it was clear from the start that the Standard Model was not complete. In particular, it left out gravity, which was the domain of Einstein's theory of general relativity. It also retained some major puzzles. E.g. quantum corrections to masses of particles due to the interactions with other particles are orders of magnitude bigger than the physical value, which should result from a delicate cancelation between basic masses and their corrections. This is often referred to as the hierarchy problem. Further there is the lack of any particle that could explain dark matter, the invisible substance that pervades the Universe.

The Standard Model is based on symmetries, then upgraded to gauge symmetries (see below in Sec. 2.1). The realization of the symmetry principle on the particle masses and interactions can take various forms. Part of the gauge symmetry may be 'spontaneously broken' by the Brout–Englert–Higgs mechanism. In the Standard Model this produces the 'unification' of weak and electromagnetic interactions. The observed strength and range of these forces are very different, yet the gauge symmetry gives them a common origin. The scale of symmetry breaking is not determined by a fundamental principle, and as such before its discovery at the LHC it was not clear at which energy one would find the Higgs particle, the essential ingredient in this mechanism.

The Standard Model symmetries, however, do not relate the two different classes of particles: bosons and fermions. The two classes are distinguished by the type of 'spins' that they possess. Fermions, which include quarks, electrons and others, have half-integer spin values such as 1/2 and 3/2. Bosons, which include photons (the particles of light), the Higgs particle, and other particles that are mediators of forces, have integer spins: 0, 1 and so on. This causes huge differences in their behavior. For fermions, the Pauli exclusion principle implies that identical particles cannot sit in the same state, while bosons can come together and join forces. The symmetries of the Standard Model do not act between these two classes. E.g. with the same set of bosons, a different set of fermions would still be compatible with the principles of the Standard Model.

In the early 1970s several researchers independently proposed [1, 2, 3] that bosons and fermions might be related to one another via a fundamental symmetry called Supersymmetry, see Figure 1 where the following developments are schematically indicated. Due to the different numbers of bosons and fermions in the Standard Model one cannot relate these already known particles. Supersymmetry rather predicts that every known particle has an as yet undetected supersymmetric partner. Bosons have a fermionic partner, e.g. the partner of the photon was baptized photino. Fermions have a bosonic partner, e.g. the partner of the electron was baptized 'selectron'. This led to a 'supersymmetric Standard Model'. The symmetric appearance of fermions and bosons, which contribute in quantum effects (loops in field theory) with opposite signs, led to a milder ultraviolet behaviour. That mechanism could explain the stability of mass differences in the Standard Model. It also led to a unification of its many undetermined coupling constants. We know that supersymmetry should, as Standard Model symmetries, be realized in a spontaneously broken way. Otherwise the partner particles should have the same mass and should already have been discovered. Unfortunately, we do not know the scale of the supersymmetry breaking. There was hope that the energies reachable by the LHC would be sufficient to see the partners of the particles of the Standard Model. Nature has not been as friendly to provide such a low scale for the spontaneous supersymmetry breaking. These particles remain hidden for now.

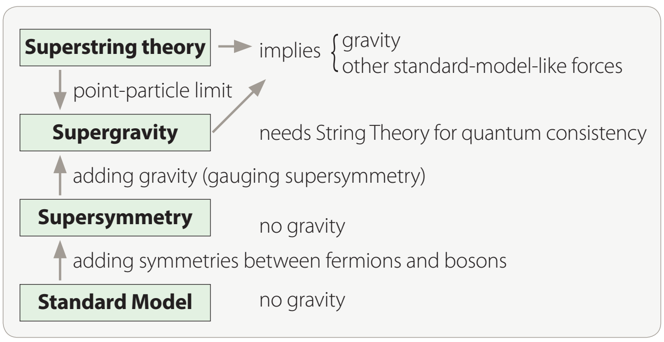

Fig. 1: The position of Supergravity in the theories of fundamental interactions.

Supersymmetry unites bosons and fermions with symmetry principles. However, it does not yet include gravity in the theory. However, in 1976 Supergravity was found by upgrading Supersymmetry to a gauge theory as we will explain in Sec. 2.4, which led to the automatic inclusion of gravity. Initial hopes that this would also remove the ultraviolet divergences in Quantum Gravity were too optimistic.

The quantum behaviour problems of a field theory are due to the fact that two particles join at a particular time in one point of space: hence in one spacetime point. If such a point is well identified, the Heisenberg uncertainty principle leads to a diverging possibility for the energy. These problems are avoided if particles are not identified with one point, but have an extension as a string. In the period mentioned above it was found that problems with tachyons (negative energy particles) in a String Theory can be avoided by inserting supersymmetry [4], defining 'superstring theory'. But soon, it was also realized that the low energy limit of superstring theory is actually Supergravity: superstring theory in the limit that the strings are reduced to points (particles) gives Supergravity theories. This limit is very often used since we know well how to calculate with particles using field theory.

Supergravity thus found a second life as a low-energy description of String Theory. The ultraviolet divergence problems of Supergravity are in this setting related to the fact that Supergravity only considers the point limit. Now Supergravity is not seen as a fundamental theory, but as a basic tool for studying applications of superstring theory.[1] The setting is exceedingly rich: it can encompass, in principle, unified models of the Fundamental Interactions. It further leads to the construction of consistent models including gravity, thereby opening a road to models for black holes and cosmology that come together with Supergravity.

[1] Some researchers do not abandon the road that Supergravity may by itself lead to a finite theory. Explicit calculations of quantum corrections to scattering amplitudes in 'extended Supergravity' (see Sec. 3.1) turn out to lead to an amazing cancelation of possible divergences, not really understood why these come about. It is not clear how far these miracles can bring us. Furthermore, these miracles do not really fit in the present picture of fundamental theories. What would we conclude if it turns out that the $\mathcal{N} = 8$ Supergravity is really finite ?

Despite the simple principles, which we will review in Sec. 2.4, underlying Supergravity, it turned out that Supergravity led to many surprises that came as an unexpected bonus. It led to a connection with very interesting geometries, exceptional symmetries and, last but not least, Supergravity contains many 'dualities', relating different theories. This played an important role in obtaining new insights in quantum field theory, and its extensions in String Theory. This goes now under the name of gauge / gravity dualities, relating classical (super)gravity theories to quantum (super)field theories.

As written in the text that announced the special breakthrough prize in fundamental physics awarded this year to the inventors of Supergravity: "Supergravity has inspired physicists for decades and may contain deep truths about the nature of reality."

2 TO THE FIRST SUPERGRAVITY THEORY

2.1 The symmetries of the Standard Model

The Standard Model is a quantum field theory that still remains the most precisely verified theory in physics. Its main theoretical progress dates from the 60s and 70s, and is based on symmetry principles. Symmetries, like rotations, are transformations characterized by parameters that describe the amount of rotation. The key theoretical concept underlying the progress in the '70s was gauge symmetry, the idea that symmetry transformations act independently at each point of spacetime. The parameters are like Lorentz scalars: arbitrary functions of the spacetime point. These parameters are coordinates of the Lie group $SU(3) \times SU(2) \times U(1)$. The fields of the theory, describing the particles, are each classified in representations of this group, and the Lagrangian is invariant under group transformations. In order to allow such a Lagrangian with symmetries independently realized at any spacetime point, the theory needs a 'gauge field' associated with every symmetry parameter. This gauge field mediates a corresponding force. As such three of the four fundamental forces of nature are associated with their own particle: the electromagnetic force is carried by the particle of light, the photon; the strong force that binds atomic nuclei is mediated by the gluon; and the weak force that governs radioactive decay is associated with W and Z particles. Despite the experimental success of verification, this model at the same time raised questions for the generation of high-energy physicists after the 70s. No fundamental principle brings us to the model with $SU(3) \times SU(2) \times U(1)$ symmetry and the presence of the known particles. Gravity finds no place in the Standard Model, and Cosmology indicates that there should be much more 'dark matter' than ordinary matter.

As mentioned in the introduction, another peculiar fact is that the symmetries do not relate bosons and fermions. Fermion fields do not have a unique value at a spacetime point: their sign is undetermined. Only expressions quadratic in fermions can get the value of a measurable quantity. Many mathematical descriptions use 'Grassmann variables' to describe fermions. Therefore the symmetry transformations of the Standard Model, determined by parameters that are scalar fields, can only relate bosons to bosons and fermions to fermions.

2.2 Supersymmetry

Then in 1973, physicists developed a principle, 'Supersymmetry' [1, 2, 3], where the parameters of the symmetry transformations are themselves fermionic. In this way the transformation of a boson field can be a quadratic expression containing the parameter and a fermionic field, such that it can change the value of a bosonic field. Vice versa the transformation of a fermion field can be the fermionic parameter times a bosonic expression, keeping its fermionic nature.

These transformations define the partners of the fields. In field theory quantum corrections are produced by the production and annihilation of virtual particles ('loop diagrams' in field theory). The calculation of these corrections often lead to divergences in integrals: they mathematically do not make sense and are a headache for the theorists who wants to use this field theory. But bosonic and fermionic virtual particles contribute with an opposite sign to these corrections. It was found that the ultraviolet divergences of supersymmetric theories are less severe than in the Standard Model due to the cancelation between bosons and fermions in loop diagrams. As such the supersymmetric Standard Model includes a mechanism explaining the tiny masses from a careful balancing act between the equal but opposite energy contributions of partner particles. It furthermore suggests the possibility that dark matter is made up of one or more of these partner particles.

The supersymmetry transformations combine in a unique framework that allows fields and particles of different spin to be unified in representations of an algebraic system called a superalgebra. Repeated supersymmetry transformations lead to symmetry under translations in spacetime, with the energy–momentum operator as conserved quantity.

Like the Standard Model, Supersymmetry initially didn't include gravity. Gravity is exceedingly weak in the subatomic domain, so that the Standard Model can attain by itself an unprecedented precision, but a fundamental theory should offer a complete picture. Gravity has a central role in the Universe at large scales. It cannot be ignored in black holes or in the early evolution of our Universe.

2.3 Quantum gravity

The fourth fundamental force, gravity, resisted efforts to include it in the model, despite the fact that Einstein's general relativity can also be formulated as a gauge theory of general coordinate transformations. The latter are translations that can be chosen independently at any spacetime point. The gauge field that corresponds to these local transformations is the graviton, a bosonic particle of spin 2 whose behaviour governs the force of gravity. The gravitational waves observed since 2015 (which was also rewarded with a Breakthrough Prize, as well as a Nobel prize) are a manifestation of the graviton field.

However, a naive merging of Quantum Mechanics and General Relativity results in a highly singular behaviour, a clash that cannot be ignored. 't Hooft and Veltman, Nobel prize winners for their understanding of the treatment of ultraviolet behaviour of the Standard Model [5] attacked the problem of the divergences in gravity [6] but found no solution. They only found a solution for the first corrections (one loop) in pure gravity, but realized that matter corrections would spoil their result. Later, it would be found that also in pure gravity the finite result of one loop is already spoiled at two loops [7]. Others followed with adding different sets of field to pure gravity but they repeatedly met with failure: the quantum corrections turned up infinities that made a nonsense of the mathematics. The aim was to find a magical model in which all the infinities would cancel. Supergravity is that special model, at least for the first few loop orders.

2.4 Supergravity

In view of the previous developments, it seems now natural to upgrade also Supersymmetry to a local symmetry. Since combining supersymmetries leads to translations, and local translations are the general coordinate transformations that are at the heart of general relativity, this brings automatically gravity in the game. That inclusion would imply that the graviton has a supersymmetric partner called the gravitino, which the theory predicts to (uniquely) have a spin of 3/2. The task to construct a Lagrangian with such a local symmetry was, however, less trivial. This was the task that Sergio Ferrara (working at CERN and INFN, Italy), Dan Freedman (long time at MIT and now in Stanford) and Peter van Nieuwenhuizen (Stony Brook University New York) [8] set their minds to while they were in Stony Brook. It involved a new interpretation of spacetime, where quantum variables, quadratic expressions in fermion (gravitino) fields, are part of the geometry of spacetime defining a torsion. In this way, Supergravity is not competing with general relativity, but rather expanding our understanding of how spacetime functions from a mathematical perspective. In fact, it took Ferrara, Freedman and van Nieuwenhuizen many months of laborious calculations. They wrote a symbolic manipulation computer program to perform the final calculation, which was too difficult to do by hand. This calculation included about 2,000 terms, all of which needed to end up being zero for their new theory of Supergravity to hold. "I sat there with mounting tension," van Nieuwenhuizen recalls. But when the program reached its conclusion successfully, he was convinced supergravity was real.

Four weeks later, Bruno Zumino, a pioneer of Supersymmetry who died in 2014, and Stanley Deser [9], now at Brandeis University, published an independent and more elegant proof that did not need any computer calculation using a 'first order' rather than 'second order' formulation of gravity.

The new ingredient, the spin 3/2 gravitino, was automatically included in the Supergravity theory with consistent interactions, while before no consistent interacting theories with spin 3/2 particles were known. This particle is the gauge field of the local supersymmetry, while the graviton is the gauge field of the general coordinate transformations. Since supersymmetry transformations combine to general coordinate transformations, these two symmetries and also their gauge fields, the graviton and gravitino, should come together. The gravitino predicted by Supergravity is a potential constituent of dark matter, which should outweigh the universes visible matter by about a factor of five.

This discovery of Supergravity in 1976 as the supersymmetric version of the theory of general relativity, opened the way to an extremely rich stream of developments in theoretical physics.

3 SUPERGRAVITY THEORIES OF VARIOUS KINDS

The first construction of pure Supergravity theory was just the beginning. In general relativity other fields like scalars, spinors and vector fields (often together called 'matter fields') are coupled to the gravitational field using a principle of minimal coupling, which means that the coupling is made consistent with symmetry under general coordinate transformations. In 1976 couplings of various supermultiplets (fields and their superpartners, e.g. spin 0 with spin 1/2 or spin 1/2 with spin 1) to pure Supergravity were obtained. This effort culminated in a rather general action of matter-coupled Supergravity in 1982 [10], which was the basis for many phenomenological models. These allowed also spontaneous supersymmetry breaking, necessary to make contact with the real world.

But there were also other extensions: as in ordinary gauge theories there are several symmetries, each with their gauge field, one could also consider several supersymmetries. Furthermore, the move from Supergravity to String Theory brings about another generalization, since Superstrings perform properly in a ten-dimensional spacetime. Therefore, when accepting this type of extension one has also to accept the presence of additional spatial dimensions that are foreign to our senses, but whose shape affects low-energy parameters via the (arbitrary) vacuum values of scalar fields, then called moduli.

3.1 Extended Supersymmetry

With one supersymmetry ($\mathcal{N} = 1$) one connects particles of spin 1/2 difference, or more exactly: states of helicity 1/2 difference. When we thus introduce 2 supersymmetries ($\mathcal{N} = 2$), the graviton will be connected not only with 2 gravitinos, but also with a particle of spin 1. Field theories (for a finite number of fields and in Minkowski space) can only be constructed for fields up to spin 2. Hence it is easy to count that, without going beyond ± 2 helicities, we can at most construct theories with 8 supersymmetries ($\mathcal{N} = 8$) connecting helicities from $+2$ to -2. This maximal $\mathcal{N} = 8$ theory [11] then contains, aside from the graviton, 8 gravitinos, 28 vectors, 56 spinors and 70 scalars. It is unique, but many shorter multiplets containing fields of different spins are available for lower values of $\mathcal{N}$. The theories are not immediately applicable for inclusion of the Standard Model. The symmetries of the Standard Model do not fit in an SO(8) group of symmetries that can be gauged by the 28 vectors. Furthermore, the observed CP violation rules out extended SUSY. This means that one should use the $\mathcal{N} = 1$ matter-coupled Supergravity in four spacetime dimensions for phenomenological purposes. This Supergravity may be derivable from extended supergravities in higher spacetime dimensions. But all these different theories have interesting properties that attracted the attention of theorists in the past years.

3.2 Higher dimensions

When we consider Supersymmetry in higher dimensions, the main feature is that the spinors have a different number of components. In 4 dimensions fermions are described by 4-component spinors, but in higher dimensions an elementary spinor has more components. The second column in Table 1 indicates the minimal number of components for a spinor in different dimensions (with Minkowski signature). For $D = 12$ we would already need 64 components. If we would write that spacetime as the product of four dimensions and consider the other 8 as very small (or consider particles that do not move in the other 8 directions), these spinors would look like $64/4 = 16$ spinors in $D = 4$. As we argued before $\mathcal{N} = 16$ is not possible for a field theory. Therefore, the highest dimension that allows a Supergravity theory is $D = 11$. This $D = 11$ theory [12] is very elegant. All the fields of the $\mathcal{N} = 8$ theory mentioned above, are put together in one graviton, one gravitino and one 3-form $A_{\mu\nu\rho}$ of $D = 11$. Table 1 gives an overview of the important supergravities. It is a simplified version of a Table that is further explained in the textbook [13].

D-dimensional spacetimes are often interpreted as the 4-dimensional spacetime of our everyday life, and a

Table 1: A map of Supersymmetry and Supergravity theories in dimensions 4 to 11. The second column gives the size of a minimal spinor (in real components) in that dimension. An entry in further columns represents the possibility to have Supergravity theories in a specific dimension D with the number of (real) supersymmetries indicated in the top row. At the bottom is indicated whether these theories exist only in Supergravity, or also with just global supersymmetry.

D	#	32	24	20	16	12	8	4
11	32	M						
10	16	IIA IIB			I			
9	16	$\mathcal{N}=2$			$\mathcal{N}=1$			
8	16	$\mathcal{N}=2$			$\mathcal{N}=1$			
7	16	$\mathcal{N}=4$			$\mathcal{N}=2$			
6	8	(2, 2)	(2, 1)		(1, 1) (2, 0)		(1, 0)	
5	8	$\mathcal{N}=8$	$\mathcal{N}=6$		$\mathcal{N}=4$		$\mathcal{N}=2$	
4	4	$\mathcal{N}=8$	$\mathcal{N}=6$	$\mathcal{N}=5$	$\mathcal{N}=4$	$\mathcal{N}=3$	$\mathcal{N}=2$	$\mathcal{N}=1$
		SG		SG/SUSY	SG	SG/SUSY		

$D - 4$ dimensional compact space. The allowed behaviour of fields by the field equations in the compact directions determine what are the different fields in 4 dimensions. This brings to full steam the elegant mathematics of Calabi–Yau spaces (6-dimensional manifolds with special properties such that they are solutions of the 10-dimensional equations) or generalizations thereof. The end result is a transcription of the internal geometry into $\mathcal{N} = 1$ or $\mathcal{N} = 2$ four-dimensional Supergravity with special combinations of matter multiplets.

An important aspect of all the theories of the table is that many more symmetries appear in the theory than the ones that were used to define or obtain them. E.g. the D =4 $\mathcal{N} = 8$ theory enjoys a E_7 symmetry, one of the large exceptional Lie algebras in the classification of simple Lie algebras. When we would further reduce to $D = 3$ we would get a theory with E_8 symmetry, the largest exceptional algebra in this classification of finite Lie algebras. For $D = 2$ and $D = 1$ infinite-dimensional algebras appear (E_9 and E_{10}), which are now studied in mathematics and physics. There is now even research on considering the theory independent of spacetime which leads to a E_{11} algebra, not much studied yet in mathematics. The lower $\mathcal{N}$ theories define special complex (Kähler) and quaternionic geometries [14, 15, 16]. This is one important example of the interplay of physics and mathematics.

4 STRING THEORY AND BEYOND

The Supergravity theories do not yet lead by themselves to a solution of the ultraviolet problem of General Relativity. This remains, by itself, the key motivation for resorting to String Theory, which is now the best hope for a theory of Quantum Gravity. Supergravity is, however, an integral piece of String Theory. It is what emerges from the theory at relatively low energies, rather like how Newtonian mechanics and gravity represent the low-energy limits of Einsteins special and general theories of relativity. If supersymmetric String Theory is correct, so is Supergravity. Many properties of String Theory can be found by considering the Supergravity limit. Actually the basic Superstring theories have as particle-physics limit the theories on the $D = 10$ line of Table 1. This was a crucial ingredient in the 1984 proof by Michael Green and John Schwarz [17] that put superstring Theory on stable mathematical footing. A few years later it led to the discovery of membranes [18], first of all coupled to the $D = 11$ supergravity, and later as crucial objects in String Theory.

4.1 Dualities and Holography

Relations between Supergravity theories (often obtained by decomposing a theory in a higher dimension to a lower dimension) have led to the discovery of dualities between Superstring theories. These remarkable generalizations of the familiar property of electric-magnetic symmetry in Maxwell Electrodynamics play a central role in the present-day picture of String Theory. Supergravity makes it possible to connect all five ten-dimensional Superstring theories to one another. As was first shown in [19, 20], the resulting picture transcends String Theory, since it also connects the ten-dimensional Superstrings to the unique eleven-dimensional version of Supergravity, where strings do not play an obvious role. The overall system is usually dubbed M-theory. It essentially uses all the extended objects that are solutions of the $D = 11$ and $D = 10$ supergravities. These contain p-branes, a generalization of membranes, which are 2-branes, or strings, 1-branes. Those solutions exist for various dimensions p and in the string picture strings can be attached to these branes.

The discovery of these dualities and earlier investigations of electric–magnetic duality in Quantum Field Theory soon converged into an unexpected spring-off. This is the AdS/CFT correspondence [21], which has endowed String Theory and Supergravity with an additional role as non-perturbative tools for a number of other research endeavors. This is a further development that started with the exact computations of the low-energy effective action for $\mathcal{N} = 2$ supersymmetric Yang—Mills theories [22, 23], with impressive consequences also in many branches of mathematics including Geometry, Topology and Group Theory. The AdS refers to Anti-de Sitter, which is a spacetime that is a solution of the gravity theory, but in this context usually a classical solution of the equations of motion of a Supergravity theory that preserves supersymmetry. The CFT on the other hand refers to a quantum field theory with conformal symmetry, mostly superconformal theory, without gravity. Meanwhile this concept has been generalized to many more cases of duality of a gauge theory and a gravity theory, connecting perturbative to non-perturbative regimes or classical to quantum computations. Usually they connect the Supergravity theory to a theory without gravity on its border (which hence has one dimension less). Relating the physics of the bulk (inner part) to a theory only at the boundary is like encoding a 3-dimensional image on a 2-dimensional picture. This is the concept of Holography and has been discovered in field theories in [24, 25, 26].

4.2 Black holes

Already in the previous section, solutions of the equations of motion of Supergravity played an important role: strings and branes are configurations that are solutions of these equations. A special case of these are black holes, which find a natural origin as solutions of the supergravity equations. They are important in Stephen Hawkings work on black hole thermodynamics. In particular they allow the study of entropy and as such played an important role in the work by Cumrun Vafa and Andrew Strominger on quantum black holes [27]. The equations that determine supersymmetric solutions are tractable first order equations, often referred to as BPS (Bogomol'nyi-Prasad-Sommerfield) solutions, easier to handle than the full second order equations of motion obtained from the Lagrangian.

5 SUPERGRAVITY FOR COSMOLOGY

Supergravity incorporates General Relativity, and its study had an important impact on our understanding of ordinary Gravity. For example, Witten [28] used a Supergravity setup in his 1981 proof of the positive energy theorem in General Relativity for which he received the prestigious Fields Medal in 1990. General Relativity apparently allows particles to have negative masses and energies, which would allow things not to fall but to raise in space. This does not happen, but was unexplained. Using the techniques of Supergravity in General Relativity enabled Witten and others to prove that particles cannot have negative masses and energies. While the proof uses Supergravity techniques, the result holds independently of whether or not Supergravity actually exists in Nature.

Cosmology appears a natural area to apply Supergravity. Comparison with our four-dimensional world are largely guided by the spontaneous breaking of local supersymmetry in $\mathcal{N} = 1$ Supergravity. This is the super Brout–Englert–Higgs mechanism, whereby a massive spin 3/2 gravitino emerges as the superpartner of the graviton. The mass of this gravitino is a supersymmetry-breaking parameter. These Supergravity models contain a potential for scalars that can include the 'inflaton', the scalar field used to describe slow-roll inflation [29, 30, 31]. Supergravity models can be built with a vacuum with negative potential energy (anti-de Sitter mentioned above for its role in AdS/CFT dualities), Minkowski vacua, which are close to our real world, or de Sitter vacua. The positive value of the potential is then the cosmological constant describing dark energy. de Sitter vacua automatically induce a spontaneous breaking of supersymmetry. Supersymmetry breaking has received considerable attention in Cosmology during the last few years, also in view of new results provided by the PLANCK satellite. The data releases have favored small values for the 'tensor–to–scalar ratio' r, which find a natural realization in the Starobinsky model [32], where the inflaton can be regarded as a purely gravitational excitation (the 'scalaron') resulting from an R^2 modification of the Einstein action by a higher-derivative term (scalar curvature squared term R^2). In recent years a constrained superfield formulation (or 'nonlinear supersymmetry') acquired considerable interest in Cosmology. In these models constraints can eliminate the supersymmetric partners of the inflaton, which otherwise introduce instabilities, leading naturally to a de Sitter inflationary phase with, at its end, supersymmetry breaking with a tiny cosmological constant. They are related to supersymmetry-breaking branes in superstring theory. Supergravity also played an instrumental role in what is known as the KKLT construction [33], which remains one of the best explanations for dark energy, the mysterious universal repulsive force pushing spacetime apart.

Other supergravity models have a running potential, often indicated as quintessence models. Since a few years a discussion is going on about criteria for Supergravity models in order that they can be included in a consistent Quantum Gravity theory, inspired by String Theory. The theories that satisfy these criteria are then indicated as the 'landscape', while for the effective Supergravity theories that do not satisfy the requirements the term 'swampland' is used [34, 35].

6 EXPERIMENTAL SEARCH FOR SUPERPARTNERS

Scientists were hoping that evidence of the superpartners of known particles might show up in the experiments at the Large Hadron Collider. However, so far these failed to uncover any signs of superpartner particles. Physicists are expanding their searches to look for nontraditional signatures of such new particles by precisely measuring known processes to see if there are inconsistencies with the predictions of the Standard Model. Planned upgrades to the LHC in the mid 2020s or a next generation of more powerful colliders in some decades will provide physicists with an order of magnitude more data to continue searching for direct signs of superpartners. Evidence for Supersymmetry could still turn up as part of current searches for dark matter and primordial gravitational waves known as B-modes.

Despite their undeniable value, there is no evidence yet that the superpartners exist in nature. At the start of the LEP accelerator in 1989 there was the hope that the Higgs particle would soon be found. It took until 2013 with the next LHC accelerator to actually discover that so-long sought particle. The present failure to find the superparticles does not, by any means, signal problems with the basic idea. This merely means that the SUSY breaking scale is considerably higher than 10 TeV and it may also take that time or longer to find them in experiments. "Nature is not being very cooperative," Freedman said in a recent interview after the announcement of the breakthrough prize. "She is hiding her secrets and not telling us the direction where the next step beyond the Standard Model lies."

For consistent ideas on the nature of fundamental particles, Cosmology, Relativity and Quantum Mechanics, Supersymmetry and Supergravity seem not just likely but virtually inescapable. Clinching evidence, though, would come from detection of the gravitino. It will be extremely hard to achieve, because the gravitino should interact very weakly with any other particle. We need to be patient. "I think it is inevitable that the spin-3/2 particle is realized in nature," Freedman says. "There is no comparable theory," Ferrara argues, "and it would be really a pity if nature has not used this one."

7 PRESENT RESEARCH

Due to the experimental state of affairs, much work has recently been done on the description of broken supersymmetry with a massive gravitino, related to anti-D3 branes in String Theory. Moreover, there are various directions in which a lot of progress has been obtained in recent years. This contains as well theories in 'low dimensions', close to our observable world, as the understanding of the structure of the more fundamental 10-or 11-dimensional theories. The progress concerns as well the structure of the theories and their symmetries and geometries, as the investigation of solutions of the equations of motion. For the latter more dualities are found to Quantum Field Theories, some defined on curved spaces, motivated by holography. The latter is also applied to exact evaluations of path integrals, denoted as 'localization' [36], and makes use of the off-shell structure of Supergravity.

Though some of these were already mentioned before, let me list here a few recent topics where progress has been made.

- The structure of the theory: new classes of gaugings have been found with a technique of embedding tensors [37, 38, 39, 40]. The gauging concerns part of the hidden symmetries that came out of the construction without really imposing them. The embedding tensor describes how the gauge vectors are connected to these symmetries, and thus which part of these symmetries are upgraded to gauge symmetries. This then also produces specific scalar potentials. Conditions on these tensors can now be solved and a classification of such gaugings is in progress. E.g. the $\mathcal{N} = 8$ supergravity in general has 912 parameters related to fluxes on branes if the $D = 4$ theory is viewed within 10-dimensional String Theory or 11-dimensional M-theory. The classification then includes a systematic study of vacua of these potentials [41]

- Models with symmetry breaking. Superfields are a way to write fields and their partners as one field in an extended space containing fermionic coordinates. Typically they describe linear supersymmetry transformations: fields transform under supersymmetry to a linear combination of partner fields. To describe supergravity with broken supersymmetry, it is advantageous to use superfields with constraints such that the transformations at the end are nonlinear [42, 43, 44]. This gives a good description of the Goldstino, the supersymmetry analogue of the Goldstone boson that goes together with symmetry breaking. Using constrained superfields, broken supersymmetry models can be systematically studied leading to de Sitter supergravity [45] and they can be connected to the theories on branes.

- Solutions of the higher-dimensional $D = 10$ or 11 Supergravity can be compactified on $\mathrm{AdS}_d \times M_{10-d}$ manifolds, i.e. with a compactified part of dimension $10 - d$. Using classification efforts or solution-generating techniques these can now be systematically studied [46].

- Consistent truncations describe D-dimensional Supergravity theories that can be obtained from a $D+d$-dimensional theory such that any solution of the lower-dimensional Supergravity is also a solution of the higher-dimensional one. In this way this lower-dimensional one is very useful as effective theory. Previously this was only obtained if the d-theory is a sphere or the manifold describing a group. With new techniques many more cases are now found [47, 48, 49, 49, 50].

- Uplifts to M-theory is the reverse way. Starting from a lower dimensional theory new variables are introduced to describe the theory as the 11-dimensional Supergravity [51]. These descriptions with new variables give a hint for a new structure, still to be discovered.

- There is much progress in the application of Supergravity for describing the black hole Bekenstein–Hawking entropy. Supergravity leads to a macroscopic description in complete agreement with the microscopic degeneracies in String Theory [52].

8 CONCLUSION

In the words of Edward Witten, an influential theoretical physicist at the Institute for Advanced Study in New Jersey who has also won a Breakthrough Prize: "Supergravity is a remarkable construction that extends Einsteins theory of gravity to include quantum variables in the structure of spacetime. Supergravity has been extraordinarily important in many later developments."

We reviewed how Supergravity started from a simple principle popular in the '70s: gauging symmetries. However, this led to many unexpected relations. Supergravity has become a powerful mathematical tool and is effectively a low-energy version of String Theory. The superpartners that are predicted are not yet found in LHC: it could be that we need much higher energies than currently accessible. Meanwhile indirect methods like precision experiments and Cosmology are used to find hints. Despite the current lack of evidence, thousands of scientists are influenced in their research by this bright idea and the obtained structures. The study of Supergravity led to a lot of insights, especially improving our understanding of Gravity. It was essential in the theoretical physics ideas for the last 40 years, with impacts on many fields of physics, including Particle Physics, Quantum Gravity, Cosmology, String Theory, and in mathematical research in Geometry, Topology and Number Theory.

Supersymmetry is alive and kicking. The formalism keeps evolving with progress in finding new models and solutions, which are important for understanding holography or black holes. It is possible that an even better formulation will still be found in the near future.

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